Does Planck mass run on the cosmological horizon scale?

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Einstein's theory of general relativity, which contains a universal value of the Planck mass, has been so far successfully invoked to explain gravitational dynamics from sub-millimeter scales to the scale of the cosmological horizon. However, one may envisage that in alternative theories of gravity, the effective value of the Planck mass (or Newton's constant), which quantifies the coupling of matter to metric perturbations, can run on the cosmological horizon scale. In this letter, we study the consequences of a glitch in the Planck mass from sub-horizon to super-horizon scales. We first give three examples of models that naturally exhibit this feature, and then show that current cosmological observations severely constrain this glitch to less than 1.2%. This is the strongest constraint to date, on *natural* (i.e. non-fine-tuned) deviations from Einstein gravity on the cosmological horizon scale.

INTRODUCTION

The Einstein theory of gravity (or General Relativity) is among the most successful theories in physics. Despite its simple mathematical structure, and having only a single constant, it has been successful in explaining the cosmological observations on the horizon scale ($\sim 10^{25-28}$cm), down to the planetary/lunar dynamics on the solar system scales ($\sim 10^{9-15}$cm), and even laboratory tests of the inverse square law on the sub-millimeter scales (see [1] for an overview).

These tests, as well as a slew of other astrophysical observations, severely constrain any alternative to the Einstein theory of gravity. Nevertheless, deviations from Einstein gravity are expected, just based on theoretical grounds. Einstein gravity is a classical theory, which does not have a well-defined quantization, and thus a more complete theory of gravity (such as string theory) is necessary to describe gravitational interaction at high energies (or small scales). However, a full theory of quantum gravity is not necessary in most models (with notable exceptions), unless we want to study interactions at very small scales (Planck length $\sim 10^{-33}$cm), which are far from the range accessible in terrestrial experiments or astrophysical processes.

Deviations from Einstein gravity have also been suggested on purely phenomenological grounds, in particular to explain the rotation curves of galaxies (as a replacement for dark matter)[2, 3], or the discovery of the apparent acceleration of cosmic expansion (as a replacement for dark energy/cosmological constant) [4, 5]. However, the evidence for any such deviation (rather than simply exotic matter/energy components), is far from conclusive.

A customary way to quantify deviations from Einstein gravity on small scales and in the weak field limit, is through introducing a Yukawa fifth force modification to the inverse square law, where the gravitational potential energy takes the form:

$$V(r) = -G\frac{m_1 m_2}{r}\left(1+\alpha e^{-r/\lambda}\right), \quad (1)$$

where G is the Newton's constant, m_1 and m_2 are the masses of (point-like) gravitating objects, while α and λ quantify the strength and the scale of the new interaction, respectively. In this model, the effective Newton's constant smoothly goes from G on large scales ($r \gg \lambda$) to $G(1+\alpha)$ on small scales ($r \ll \lambda$). Current experimental and observational constraints severely limit α in the range $10^{-1}\text{cm} < \lambda < 10^{16}$cm (see [1] for an overview).

In this letter, we investigate the possibility of a similar glitch in the Newton's constant (or Planck mass) on the scale of the cosmological horizon, or the Hubble radius ($\lambda \sim c/H \sim 10^{28}$cm). In our case, the scale λ will not be a physical constant of the theory, but rather an emergent scale in the theory, as a consequence of an effective change in the background geometry, from flat Minkowski space on small scales, to the expanding Friedmann-Robertson-Walker background on large scales.

We start by defining an effective Planck mass, and then give a few examples of the theories which contain a glitch in their effective Planck masses on the horizon scale. We will then investigate the cosmological consequences of such a glitch for structure formation, and the cosmic microwave background, and provide a limit based on current cosmological observations.

EFFECTIVE PLANCK MASS

The Planck mass, M_p, quantifies the strength of coupling between the space-time metric, and the energy-momentum of matter in the Universe. In terms of the Einstein equation:

$$G^\mu_\nu = M_p^{-2} T^\mu_\nu, \quad (2)$$

where G^μ_ν and T^μ_ν are the Einstein and the total energy-momentum tensors respectively. Notice that in our notation, $M_p = (8\pi G)^{-1/2} \simeq 2.44 \times 10^{18}$GeV, where G is Newton's gravitational constant, and we have used natural units ($\hbar = c = 1$).

Even though the Planck mass is a constant of the Einstein theory of gravity, possible deviations from Einstein gravity, or alternatively, other energy components that are not accounted for in the total energy momentum tensor, $T_{\mu\nu}$, may lead to an effective (or dressed) Planck mass that could run with time and/or the energy/length scale of the interactions. A possible definition for an effective Planck mass may come by perturbing the Einstein constraint (or G^0_0) equation:

$$M^{-2}_{p,\rm eff} \equiv \frac{\delta G^0_0}{\delta T^0_0}, \tag{3}$$

which reduces to the Poisson equation around a Minkowski background (or on sub-horizon scales). However, Eq. (3) can mix different scales, as it involves the ratio of two variable functions. Moreover, this definition may become ill-defined if δT^0_0 crosses zero. Instead, we are going to adopt a more practical definition:

$$M^{-2}_{p,\rm eff}(|\mathbf{k}|) \equiv \frac{\langle \delta G^0_{0,\mathbf{k}} \delta T^{0*}_{0,\mathbf{k}}\rangle}{\langle \delta T^0_{0,\mathbf{k}} \delta T^{0*}_{0,\mathbf{k}}\rangle}, \tag{4}$$

where $\delta T^0_{0,\mathbf{k}}$ and $\delta G^0_{0,\mathbf{k}}$ are the spatial Fourier transforms of δT^0_0 and δG^0_0 on a given spatial hypersurface.

While this definition has the benefit of separating different physical scales, we have introduced an explicit gauge-dependence through the choice of a particular spatial hypersurface. On small (sub-horizon) scales ($k \gg H$), this gauge dependence is not important, as Eq. (3) reduces to the Poisson equation, and we recover Newtonian gravity. In other words, the difference between $M_{p,\rm eff}$ in different (physical) gauges is $\sim (k/H)^{-2}$, on sub-horizon scales.

On super-horizon scales ($k \ll H$), gauge transformations can change the effective Planck mass, only if the Planck mass associated with the background expansion is different from the Planck mass associated with the perturbations, i.e. as long as:

$$M^{-2}_{p,\rm IR} = \frac{\dot{G}^0_0}{\dot{T}^0_0} = \frac{\delta G^0_0}{\delta T^0_0}, \tag{5}$$

the effective Planck mass is gauge-invariant on super-horizon scales. This condition can naturally result from the assumption of adiabatic initial conditions, which asserts that, up to a time shift, causally disconnected patches of the Universe experience identical histories. Therefore, we see that, at least for *adiabatic* initial conditions, the gauge dependence of our definition of the effective Planck mass may only become important as modes cross the horizon.

From here on, we will refer to the cosmological sub-horizon ($k \gg H$) and super-horizon ($k \ll H$) scales as the UV and IR scales, respectively, which have their respective values of the effective Planck mass, $M_{p,\rm UV}$ and $M_{p,\rm IR}$. In the language of Eq. (1), the UV-IR mismatch can be parametrized by the dimensionless α parameter:

$$M^2_{p,\rm IR} = M^2_{p,\rm UV}(1+\alpha). \tag{6}$$

THREE EXAMPLES

As an example, let us consider the quadratic Cuscuton action [6]:

$$S_Q = \int d^4x\sqrt{-g}\left(\mu^2\sqrt{|\partial^\mu\varphi\partial_\mu\varphi|} - \frac{1}{2}m^2\varphi^2\right), \tag{7}$$

where φ is a scalar field, and μ and m are constants of theory with the dimensions of energy.

If we consider Cuscuton as a part of the gravitational action (and so do not include it in the energy-momentum tensor), the effective Planck mass takes the form:

$$M^{-2}_{p,\rm eff}(k) = \frac{\delta\rho_Q + \delta\rho_m}{\delta\rho_m}. \tag{8}$$

Using the solution to the field equation in the Longitudinal gauge, obtained in [7], we find that:

$$M^2_{p,\rm eff} \simeq M^2_{p,\rm UV} - \frac{3\mu^4}{2m^2}\left(1+\frac{k^2}{3H^2}\right)^{-1}\left(1-\frac{k^2}{3\dot{H}}\right)^{-1}, \tag{9}$$

to the lowest order in μ, in a flat matter-dominated Universe.

As we noted above, the exact k-dependence of $M_{p,\rm eff}$ will depend on the choice of gauge, but the IR limit of the effective Planck mass:

$$M^2_{p,\rm IR} = M^2_{p,\rm UV} - \frac{3\mu^4}{2m^2}, \tag{10}$$

is set by the Friedmann equation [6], and is thus gauge-invariant [19].

A very similar behavior can be seen in the dynamics of a canonical scalar (or quintessence) field with a simple exponential potential [8]:

$$V(\varphi) = M^4_p e^{-\kappa\varphi/M_p}. \tag{11}$$

It is easy to see that, for a fixed background equation of state, the energy density of the field, asymptotically, reaches a constant fraction of the energy density of the Universe. In particular, for a flat matter-dominated cosmology, this fraction is:

$$\Omega_\varphi = \frac{3}{\kappa^2}, \tag{12}$$

which translates to a glitch in the effective Planck mass on the horizon scale:

$$M_{p,\mathrm{IR}}^2 = M_{p,UV}^2\left(1-\frac{3}{\kappa^2}\right), \tag{13}$$

as quintessence does not cluster on sub-horizon scales, and so $M_{p,\mathrm{eff}}$ reaches its fundamental value on small scales.

A third model that leads to a similar mismatch between the IR and UV effective Planck masses has been introduced by Carroll and Lim [9], and consists of a Lorentz-violating, fixed-norm, time-like vector field u^μ, with the Lagrangian:

$$\begin{aligned}\mathcal{L}_u &= -\beta_1\nabla^\mu u^\sigma\nabla_\mu u_\sigma - \beta_2\left(\nabla_\mu u^\mu\right)^2 - \beta_3\nabla^\sigma u^\mu\nabla_\mu u_\sigma \\ &+ \lambda_u(u^\mu u_\mu + m_u^2),\end{aligned} \tag{14}$$

where m_u is the norm of the vector field, λ_u is a Lagrange multiplier, and β_i's are dimensionless constants of the theory. In this model, the coupling of u^μ to the metric renormalizes both the IR and UV values of the effective Planck mass, so that:

$$M_{p,\mathrm{IR}}^2 = M_{p,\mathrm{UV}}^2 + (2\beta_1+3\beta_2+\beta_3)m_u^2. \tag{15}$$

It is interesting to notice that, unlike the two scalar field models discussed before, the gravitational force is suppressed on large (super-horizon) scales by the Lorentz-violating vector field [9].

We will next look at the observational consequences of a possible glitch between the UV and IR effective Planck masses.

COSMOLOGICAL CONSTRAINTS ON THE RUNNING OF THE PLANCK MASS

The Hubble expansion rate in a flat homogenous cosmology is set by the Friedmann equation:

$$H^2 = \frac{\rho_{\mathrm{tot}}}{3M_{p,\mathrm{IR}}^2}. \tag{16}$$

As the present-day cosmic density, ρ_{tot}, is dominated by dark matter and dark energy, which are only seen through their gravitational effects, there is no way to find $M_{p,\mathrm{IR}}$ through measuring the present-day Hubble constant. However, the energy density in the radiation era is dominated by photons and neutrinos (for $T \lesssim 1$ MeV), which are better understood, as their energy density (for three species of relativistic neutrinos) is fixed by the Cosmic Microwave Background (CMB) temperature ($T = 2.728 \pm 0.004$ K, [11]). Constraints on the expansion rate during the radiation era (at $T \sim 0.1$ MeV) then come from comparing the Big Bang Nucleosynthesis predictions with the cosmological observations of the light element abundances, which correspondingly constrain the running of the Planck mass: $\alpha = 0.0 \pm 0.2$ (95% confidence level) [12].

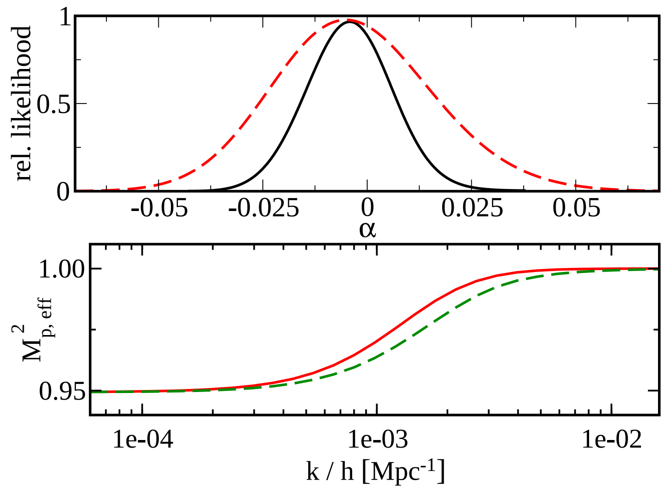

FIG. 1: *Top Panel:* Observational constraints on the UV/IR Planck mass mismatch parameter, α, from 3 years of WMAP data alone [10](red, dashed line), and our compilation (see the text) of cosmological observations (black straight line). *Bottom Panel:* Transition between the IR and UV regimes for the effective Planck mass (in units of $M_{p,\mathrm{IR}}$) defined in Eq.(8) for the quadratic Cuscuton with $\alpha = -0.05$ (red, straight line). The transition for a canonical scalar field model ($c_s^2 = 1$) is depicted in green (dashed line). For $c_s^2 \gtrsim 10$, the transitions virtually coincide with the quadratic Cuscuton (for which $c_s^2 = \infty$).

More interesting constraints can come from the study of cosmological perturbations on small scales. Combining the continuity and Poisson equations with Newton's 2nd law yields:

$$\ddot\delta + 2H\dot\delta = \frac{\bar\rho_m}{2M_{p,\mathrm{UV}}^2}\delta, \tag{17}$$

for the linear matter overdensity perturbations δ ($= \delta\rho_m/\bar\rho_m$) on small scales. Combining this with the Friedmann equation (Eq. 16) in the matter-dominated era, and the definition of α (Eq. 6), we find:

$$\ddot\delta + 2H\dot\delta - \frac{3}{2}(1+\alpha)H^2\delta = 0, \tag{18}$$

which can be easily solved (using $H = 2/3t$, in the matter-dominated era). For $\alpha \ll 1$, the growing mode behaves as:

$$\delta \propto t^{\frac{2}{3}+\frac{2}{5}\alpha} \Rightarrow \Phi \propto t^{\frac{2}{5}\alpha}, \tag{19}$$

where Φ is the Newtonian (or longitudinal metric) potential.

Perturbation modes that are inside the horizon at the time of matter-radiation equality will then all experience the same amount of suppression or enhancement during the matter era. Since the scale factor grows as $t^{2/3}$,

this suppression/enhancement is roughly by a factor of $z_{\rm eq}^{3\alpha/5} \simeq 1+5\alpha$, where $z_{\rm eq} \simeq 3400$ is the redshift of matter-radiation equality.

For the angular spectrum of CMB anisotropies, this will result in a small change in the power on small scales, and also in a change of the contribution from the Integrated Sachs-Wolfe (ISW) effect, as a result of the decaying/growing Newtonian potential [20]. For $\alpha > 0$, there will be less power on the large scale CMB power spectrum, whereas a negative α will lead to an increase of power. Correspondingly, the acoustic peak of the CMB power spectrum will be slightly shifted, due to the change in the cosmic expansion history (see Fig.2 in [7], in which $\Omega_Q = -\alpha$). Therefore, a possible running of the Planck mass on the horizon scale can be constrained by CMB observations. We compute these constraints using a modified version of cmbeasy [13] for the quadratic Cuscuton model (see [7] for details). We find that the 3-year CMB power spectrum of WMAP [10] constrains α to -0.005 ± 0.040 (at 95% confidence).

The impact of a UV/IR glitch in the effective Planck mass on structure formation could be equally significant. The most prominent effect is the change in the amplitude of the matter power spectrum (in comparison to the CMB power) on small scales at late times. In addition, modes entering the horizon at different times will be suppressed or enhanced (depending on the sign of α) by a factor which depends on the time when they enter the horizon, as we have seen above. As a result, the cold dark matter power spectrum will also be tilted between the equality and present-day horizon scales.

Hence, the amplitude of a possible UV/IR mismatch of the effective Planck mass can be also constrained by observations of large scale structure. We use the latest data from the distribution of luminous red galaxies from the Sloan Digital Sky Survey (SDSS) [14] (marginalizing over bias). We also include constraints on the cold dark matter power spectrum from observations of the Lyman-α forest [15]. Even though this data extends into the mildly non-linear regime of the power spectrum, we expect non-linear effects (of a non-vanishing α) to be of little importance here, in particular because the bounds on α are already rather tight from CMB alone. Adding the results from Supernovae Ia observations [16] as well as the observation of the baryon acoustic oscillations (BAO) [17] to this large scale structure data (in order to decrease degeneracy with other cosmological parameters), and the data from 3 years of WMAP [10], we find the UV/IR mismatch parameter, α, to be tightly constrained to -0.004 ± 0.021 (95% CL) by our complete set of current observational data (see Fig. 1).

DISCUSSIONS

One may wonder if our constraints on α may depend on the specific model that yields the running of the effective Planck mass. Fig. 1 compares the UV-IR transition of $M^2_{p,{\rm eff}}$ (in longitudinal gauge) for the quadratic Cuscuton and the exponential scalar field models in the matter era. We see that for the canonical scalar field model the transition is shifted to slightly smaller scales by $\sim 30\%$. However, the effect on the bounds on a UV/IR glitch that were computed above is marginal. For the scalar field model with $c_s^2 = 1$, the bounds on α are only slightly relaxed, namely to -0.004 ± 0.024 at the 95% confidence level. Therefore, we conclude that the constraints on α are insensitive to the details of UV/IR transition, since most of the observable consequences of the mismatch occur on small sub-horizon scales.

What about an arbitrary redshift evolution of the running factor, $\alpha(z)$? While this is, in principle possible, and in line with the current efforts to quantify the redshift evolution of dark energy, we should point out that any such evolution would require introducing an *ad hoc* macroscopic scale (coincident with the present-day horizon) into the theory. Indeed, this is the same fine-tuning problem that most dark energy or modified gravity theories (with non-trivial late-time dynamics) suffer from. In lieu of any such scale, the cosmological horizon is the only macroscopic scale in the problem that could control the running of gravitational coupling constants. Therefore, a constant α is the only natural result of a non-trivial *microscopic* physics in the gravity theory.

To summarize, in this letter, we have studied the running of the Planck mass (or Newton's constant) on the cosmological horizon scale, as a possible modification of Einstein gravity. We have first discussed three physical models that naturally exhibit this running. We then considered observable consequences of this running and found out that any mismatch between UV and IR Planck masses (Newton's constants) is severely constrained to less than 1.2% (2.4%) at the 95% confidence level. While future cosmological observations are likely to strengthen this bound by an order of magnitude over the next decade, the expected magnitude of such a running/glitch in well-motivated extensions to Einstein gravity is yet to be determined.

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[19] In fact, Eq. (10) holds exactly for quadratic Cuscuton, independent of the equation of state of the rest of the energy components of the Universe.
[20] Presence of anisotropic stress in some modifications to Einstein gravity, such as the Lorentz-violating vector field, can change the predictions for the ISW effect. However, given the current constraints on the anisotropic stress [18], this is unlikely to affect our results significantly.